Is Mathematics Useful?

David Owen
County Advisor for
Primary Mathematics

Throughout history there has been a conflict between mathematics seen as a subject growing out of economic and social necessity and the view that mathematics has a purity which transcends mere practicality. Euclid, when asked by a slave what he would get by learning geometry, offered him a coin of low value so that he could gain from what he learned! More recently, G.H. Hardy, in 'A Mathematician's Apology' (Cambridge, 1948), said, "I have never done anything useful. No discovery of mine has been made, or is likely to make, directly, or indirectly, for good or ill, the least difference to the amenity of the world." A contrary position was taken by Robert Recorde writing in 1540 in which he suggests that:

"If number be lacking it maketh men dumbe
So that to most questions they must answer NUM"

To a certain extent these three great mathematicians miss the point - mathematical knowledge exists within a social and cultural context. Thus, much of Robert Recorde's arithmetical work became less useful with the development of logarithms in the 18th century, and totally redundant with the advent of modern calculating devices. In contrast, Hardy's work does have practical applications in the modern world and may have even more powerful influences in the future.

It is inappropriate in this brief paper to consider the nature of mathematics in any detail - suffice it to say that the subject is made by men and exists only in their minds. The power of the subject, in fact, lies in this abstract nature, which enables structures to be applied in a variety of contexts. Sometimes this application is concerned with exploring and explaining the natural and man-made world. A simple numerical sequence developed by Leonardo of Pisa (Fibonacci) in the 13th century has links with, amongst other things, the Acropolis, the shape of writing paper, snail shells, flower petals, the paintings of Leonardo da Vinci and the architectural work of Le Corbusier! In a more ordinary context, all of us form simple mathematical models to help us deal with everyday happenings. For example, given that I know that 3 x 2 = 6, I can use this when I am buying: 3 boxes of chocolates at #2 a box, 20 metres of material at #3 a metre, and 3 metres of material at #20 a metre. I can also estimate the cost of 3 gallons of petrol at #1.87 a gallon and how far I will travel in two hours at approximately 30 m.p.h. Although I would not wish to subscribe to the Pythagorean view of mathematics as magic, the examples given show how the former can intrude into an area normally seen as the province of the latter. They could all be concerned with prediction. In the end, it is only mathematics that can provide us with data that enables us to predict the safety of a suspension bridge. It is only mathematics that can save Venice from a watery grave.

How, then, do these issues relate to the mathematical education of young children? Two major corner stones of modern educational thought are that pupils must understand the work they are tackling and be able to make effective application of their knowledge. This is well expressed in the Cockcroft Report ('Mathematics Counts', HMSO, 1982) as follows:

> "Most important of all is the need to have sufficient confidence to make effective use of whatever skill and understanding is needed - whether this be little or much."

To assist understanding it is now firmly established that mathematics, that most abstract of subjects, must be learned in a practical way, involving the use of first-hand experience relevant to the child. For example, earlier this century Dienes developed an elegant analogy to our system of writing numbers using blocks of wood. These are now widely used in schools to assist children to grasp the complexities of the system.

Other important concerns are to provide pupils with the opportunity to investigate, solve problems and to make discoveries in mathematics. The Association of Teachers of Mathematics argued in 'Notes on Mathematics in Primary Schools' (Cambridge, 1967) that there is no clear distinction between a mathematician inventing new mathematics and a child learning mathematics which is new to him. In this sense, mathematics can only be learnt by being created or re-created.

It is best, perhaps, to finish with two examples which demonstrate these points with the additional thought that mathematics is best learnt in a group setting through interaction with others. The first example is concerned with a possible way of involving a group of children in an activity leading towards the establishment of Euler's Formula. In 1735 Euler proved that for every polyhedron (a solid which is bounded by flat surfaces only - such as a cube or a pyramid) the sum of the number of faces and vertices is always two more than the number of edges (an edge is the straight line where two faces meet and a vertex is the corner where a number of edges meet). This can be expressed symbolically as:

$$V + F = E + 2$$

This is probably quite boring to most people when given as a piece of straight information. However, enormous excitement can be created in children if:

1. They are allowed to handle a fairly large number of polyhedra (say 20) and discuss them with other children. They may have been involved previously in making some of these polyhedra.

2. They are invited to record in a table the number of vertices, faces and edges for each shape.
3. They are asked to discuss with their friends any patterns in the table.

4. They are asked to forecast (say) the number of edges in a shape given the number of faces and the number of vertices.

In most cases in a class of ten or eleven year olds some pupils will discover Euler's relationship in one form or another. Enormous satisfaction can be obtained through a rediscovery of a mathematical relationship first developed by a master mathematician. Similar approaches could be used to rediscover, for example, the probability theories of Blaise Pascal, the French mathematician who was highly motivated by his desire to win at the card table!

My second example relates to a classroom incident, where a six year old actually invented his own mathematics. I was talking to a group of six year olds who were working in a classroom shop when (for some bizarre reason) it became necessary to subtract six from two. I asked the children, "What do you think 2 - 6 might be?" Almost all of them responded very quickly with, "Four!" and then equally quickly with, "Oh, no, it's not, is it!?" I was just beginning to regret asking such a barmy question when one six year old said, "I think it's 0, 0, 0, 0, 0!" Some might think that a silly question deserves a silly answer, but in fact the boy concerned made a huge intellectual jump. He realised that the numbers that he knew were inadequate in relation to the problem and it was therefore necessary to invent a new system of numbers for answering such questions. He knew that 2 - 2 = 0 and invented his system which would give:

2 - 3 = 0, 0
2 - 4 = 0, 0, 0
2 - 5 = 0, 0, 0, 0
2 - 6 = 0, 0, 0, 0, 0